Calculus 1
Workbook Solutions

Chain rule

krista king
MATH

WITH POWER RULE

■ 1. Find $h'(x)$ if $h(x) = (3x^2 - 7)^4$.

Solution:

To find the derivative, we have to apply chain rule. We'll say that the inside function is $3x^2 - 7$, and that the derivative of that inside function is $6x$. Therefore, the derivative is

$$h'(x) = 4(3x^2 - 7)^3(6x)$$

$$h'(x) = 24x(3x^2 - 7)^3$$

■ 2. Find $h'(x)$ if $h(x) = 2(5x^2 + 2x)^3$.

Solution:

To find the derivative, we have to apply chain rule. We'll say that the inside function is $5x^2 + 2x$, and that the derivative of that inside function is $10x + 2$. Therefore, the derivative is

$$h'(x) = 6(5x^2 + 2x)^2(10x + 2)$$

$$h'(x) = 6(10x + 2)(5x^2 + 2x)^2$$

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■ 3. Find $h'(x)$ if $h(x) = (2x^2 - 6x + 5)^7$.

Solution:

To find the derivative, we have to apply chain rule. We'll say that the inside function is $2x^2 - 6x + 5$, and that the derivative of that inside function is $4x - 6$. Therefore, the derivative is

$$h'(x) = 7(2x^2 - 6x + 5)^6(4x - 6)$$

$$h'(x) = 7(4x - 6)(2x^2 - 6x + 5)^6$$

■ 4. Find $h'(x)$ if $h(x) = 2(x^3 + 4x^2 - 2x)^5$.

Solution:

To find the derivative, we have to apply chain rule. We'll say that the inside function is $x^3 + 4x^2 - 2x$, and that the derivative of that inside function is $3x^2 + 8x - 2$. Therefore, the derivative is

$$h'(x) = 2(5)(x^3 + 4x^2 - 2x)^4(3x^2 + 8x - 2)$$

$$h'(x) = 10(3x^2 + 8x - 2)(x^3 + 4x^2 - 2x)^4$$

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WITH PRODUCT RULE

■ 1. Find $y'(x)$ if $y(x) = (3x - 2)(5x^3)^5$.

Solution:

To find the derivative, we have to apply product rule.

$$y'(x) = \frac{d}{dx}(3x - 2) \cdot (5x^3)^5 + (3x - 2) \cdot \frac{d}{dx}(5x^3)^5$$

To find each derivative, we have to apply chain rule.

$$y'(x) = 3 \cdot (5x^3)^5 + (3x - 2) \cdot 5(5x^3)^4(15x^2)$$

$$y'(x) = 3(5x^3)^5 + 75x^2(3x - 2)(5x^3)^4$$

■ 2. Find $h'(x)$ if $h(x) = (x^2 - 5x)^2(2x^3 - 3x^2)^5$.

Solution:

To find the derivative, we have to apply product rule.

$$h'(x) = \frac{d}{dx}(x^2 - 5x)^2 \cdot (2x^3 - 3x^2)^5 + (x^2 - 5x)^2 \cdot \frac{d}{dx}(2x^3 - 3x^2)^5$$

To find each derivative, we have to apply chain rule.

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$$h'(x) = 2(x^2 - 5x)(2x - 5) \cdot (2x^3 - 3x^2)^5 + (x^2 - 5x)^2 \cdot 5(2x^3 - 3x^2)^4(6x^2 - 6x)$$

$$h'(x) = 2(2x - 5)(x^2 - 5x)(2x^3 - 3x^2)^5 + 5(6x^2 - 6x)(x^2 - 5x)^2(2x^3 - 3x^2)^4$$

■ 3. Find $h'(x)$ if $h(x) = (x + 4)^5(3x - 2)^3$.

Solution:

To find the derivative, we have to apply product rule.

$$h'(x) = \frac{d}{dx}(x + 4)^5 \cdot (3x - 2)^3 + (x + 4)^5 \cdot \frac{d}{dx}(3x - 2)^3$$

To find each derivative, we have to apply chain rule.

$$h'(x) = 5(x + 4)^4(1) \cdot (3x - 2)^3 + (x + 4)^5 \cdot 3(3x - 2)^2(3)$$

$$h'(x) = 5(x + 4)^4(3x - 2)^3 + 9(x + 4)^5(3x - 2)^2$$

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WITH QUOTIENT RULE

■ 1. Find $h'(x)$.

$$h(x) = \frac{(2x+1)^3}{(3x-2)^2}$$

Solution:

To find the derivative, we have to apply quotient rule.

$$h'(x) = \frac{\frac{d}{dx}(2x+1)^3 \cdot (3x-2)^2 - (2x+1)^3 \cdot \frac{d}{dx}(3x-2)^2}{((3x-2)^2)^2}$$

To find each derivative, we have to apply chain rule.

$$h'(x) = \frac{3(2x+1)^2(2) \cdot (3x-2)^2 - (2x+1)^3 \cdot 2(3x-2)(3)}{((3x-2)^2)^2}$$

$$h'(x) = \frac{6(2x+1)^2(3x-2)^2 - 6(2x+1)^3(3x-2)}{(3x-2)^4}$$

$$h'(x) = \frac{6(2x+1)^2(3x-2) - 6(2x+1)^3}{(3x-2)^3}$$

Factor the numerator.

$$h'(x) = \frac{6(2x+1)^2(3x-2-(2x+1))}{(3x-2)^3}$$

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$$h'(x) = \frac{6(2x+1)^2(3x-2-2x-1)}{(3x-2)^3}$$

$$h'(x) = \frac{6(2x+1)^2(x-3)}{(3x-2)^3}$$

2. Find $h'(x)$.

$$h(x) = \frac{(4x+5)^5}{(x+3)^2}$$

Solution:

To find the derivative, we have to apply quotient rule.

$$h'(x) = \frac{\frac{d}{dx}(4x+5)^5 \cdot (x+3)^2 - (4x+5)^5 \cdot \frac{d}{dx}(x+3)^2}{((x+3)^2)^2}$$

To find each derivative, we have to apply chain rule.

$$h'(x) = \frac{5(4x+5)^4(4) \cdot (x+3)^2 - (4x+5)^5 \cdot 2(x+3)(1)}{(x+3)^4}$$

$$h'(x) = \frac{20(4x+5)^4(x+3)^2 - 2(4x+5)^5(x+3)}{(x+3)^4}$$

$$h'(x) = \frac{20(4x+5)^4(x+3) - 2(4x+5)^5}{(x+3)^3}$$

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Factor the numerator.

$$h'(x) = \frac{2(4x+5)^4(10(x+3)-(4x+5))}{(x+3)^3}$$

$$h'(x) = \frac{2(4x+5)^4(10x+30-4x-5)}{(x+3)^3}$$

$$h'(x) = \frac{2(4x+5)^4(6x+25)}{(x+3)^3}$$

3. Find $h'(x)$.

$$h(x) = \frac{(7x-4)^3}{(5x+3)^2}$$

Solution:

To find the derivative, we have to apply quotient rule.

$$h'(x) = \frac{\frac{d}{dx}(7x-4)^3 \cdot (5x+3)^2 - (7x-4)^3 \cdot \frac{d}{dx}(5x+3)^2}{((5x+3)^2)^2}$$

To find each derivative, we have to apply chain rule.

$$h'(x) = \frac{3(7x-4)^2(7) \cdot (5x+3)^2 - (7x-4)^3 \cdot 2(5x+3)^1(5)}{(5x+3)^4}$$

$$h'(x) = \frac{21(7x-4)^2(5x+3)^2 - 10(7x-4)^3(5x+3)}{(5x+3)^4}$$

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$$h'(x) = \frac{21(7x-4)^2(5x+3) - 10(7x-4)^3}{(5x+3)^3}$$

Factor the numerator.

$$h'(x) = \frac{(7x-4)^2(21(5x+3) - 10(7x-4))}{(5x+3)^3}$$

$$h'(x) = \frac{(7x-4)^2(105x+63 - 70x+40)}{(5x+3)^3}$$

$$h'(x) = \frac{(7x-4)^2(35x+103)}{(5x+3)^3}$$

4. Find $h'(x)$.

$$h(x) = \frac{(6x-1)^4}{(8x+1)^2}$$

Solution:

To find the derivative, we have to apply quotient rule.

$$h'(x) = \frac{\frac{d}{dx}(6x-1)^4 \cdot (8x+1)^2 - (6x-1)^4 \cdot \frac{d}{dx}(8x+1)^2}{((8x+1)^2)^2}$$

To find each derivative, we have to apply chain rule.

$$h'(x) = \frac{4(6x-1)^3(6) \cdot (8x+1)^2 - (6x-1)^4 \cdot 2(8x+1)(8)}{(8x+1)^4}$$

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$$h'(x) = \frac{24(6x-1)^3(8x+1)^2 - 16(6x-1)^4(8x+1)}{(8x+1)^4}$$

$$h'(x) = \frac{24(6x-1)^3(8x+1) - 16(6x-1)^4}{(8x+1)^3}$$

Factor the numerator.

$$h'(x) = \frac{8(6x-1)^3(3(8x+1) - 2(6x-1))}{(8x+1)^3}$$

$$h'(x) = \frac{8(6x-1)^3(24x+3-12x+2)}{(8x+1)^3}$$

$$h'(x) = \frac{8(6x-1)^3(12x+5)}{(8x+1)^3}$$

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WITH TRIG FUNCTIONS

■ 1. Find $f'(x)$ if $f(x) = -5\cos^2(3x - 4)$.

Solution:

Rewrite the function so that we can see it clearly as a power function.

$$f(x) = -5(\cos(3x - 4))^2$$

Use power rule to find the derivative, remembering to apply chain rule.

$$f'(x) = -5(2)(\cos(3x - 4))^1(-\sin(3x - 4))(3)$$

$$f'(x) = -30(\cos(3x - 4))(-\sin(3x - 4))$$

$$f'(x) = 30\sin(3x - 4)\cos(3x - 4)$$

■ 2. Find $g'(x)$ if $g(x) = 6\sin^3(5x + 2)$.

Solution:

Rewrite the function so that we can see it clearly as a power function.

$$g(x) = 6(\sin(5x + 2))^3$$

Use power rule to find the derivative, remembering to apply chain rule.

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$$g'(x) = 6(3)(\sin(5x+2))^2(\cos(5x+2))(5)$$

$$g'(x) = 90(\sin(5x+2))^2(\cos(5x+2))$$

$$g'(x) = 90\sin^2(5x+2)\cos(5x+2)$$

■ 3. Find $h'(x)$ if $h(x) = \tan^3(7x^2+3x)$.

Solution:

Rewrite the function so that we can see it clearly as a power function.

$$h(x) = (\tan(7x^2+3x))^3$$

Use power rule to find the derivative, remembering to apply chain rule.

$$h'(x) = 3(\tan(7x^2+3x))^2(\sec^2(7x^2+3x))(14x+3)$$

$$h'(x) = 3(14x+3)\tan^2(7x^2+3x)\sec^2(7x^2+3x)$$

■ 4. Find $k'(x)$ if $k(x) = 3\sin^2(2x^2+4x-9)$.

Solution:

Rewrite the function so that we can see it clearly as a power function.

$$k(x) = 3(\sin(2x^2+4x-9))^2$$

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Use power rule to find the derivative, remembering to apply chain rule.

$$k'(x) = 3(2)(\sin(2x^2 + 4x - 9))^1(\cos(2x^2 + 4x - 9))(4x + 4)$$

$$k'(x) = 6(4x + 4)\sin(2x^2 + 4x - 9)\cos(2x^2 + 4x - 9)$$

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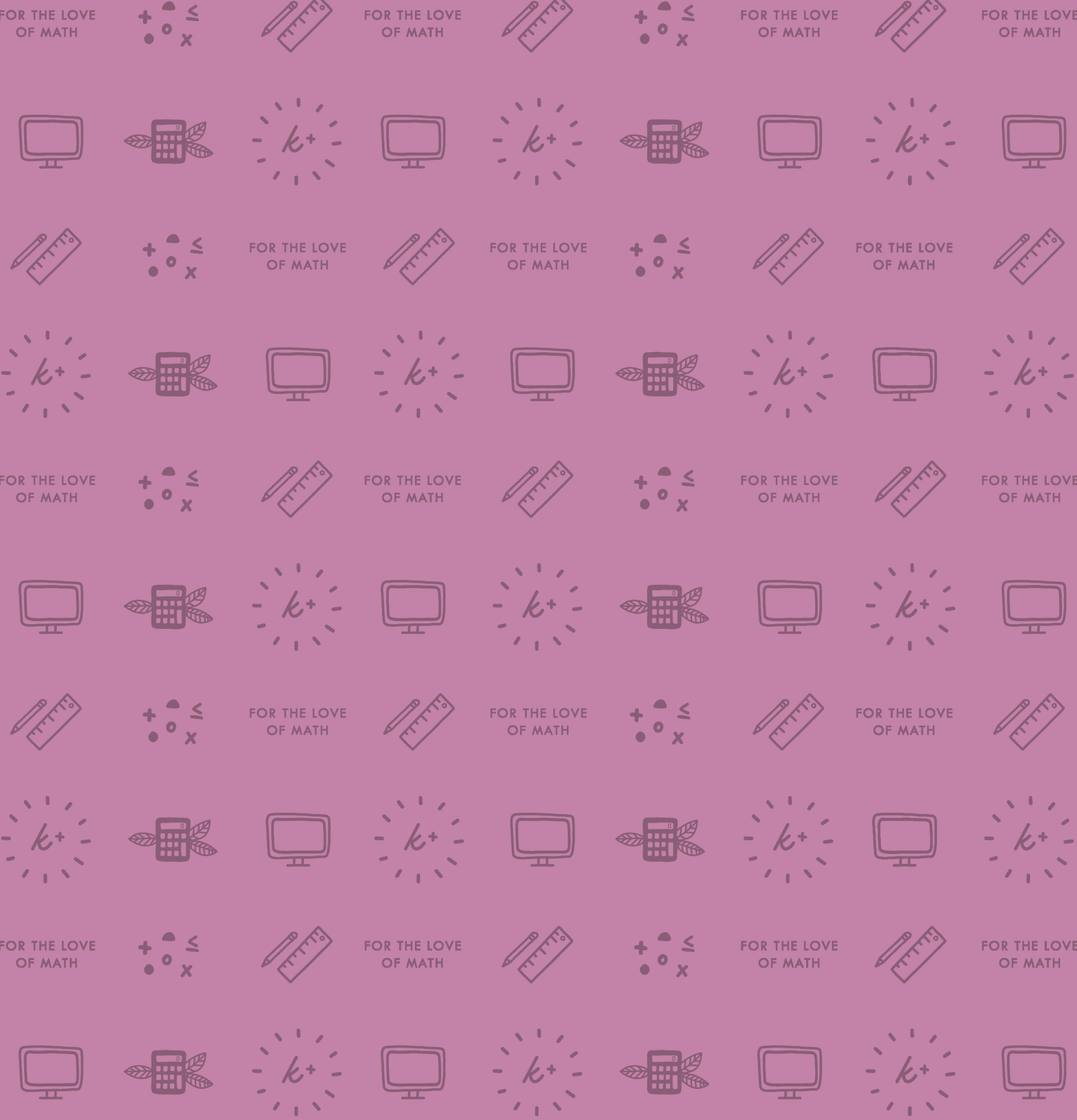

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